Minimum-Time Rescue Trajectories between Spacecraft in Circular Orbits

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Optimal space trajectories to be flown by a rescue vehicle so as to rendezvous with a distressed spacecraft in minimum time are investigated. Nonlinear equations describing the inverse square gravitational field are used, and large transfer angles are considered. The rocket thrust of the rescue vehicle is allowed to vary between zero and a specified upper limit. The distressed vehicle is passive. Both vehicles are initially in circular orbits, but not necessarily at the same altitude nor in the same plane. Atmospheric effects are neglected, but the minimum altitude of the rescue craft is restricted by introducing a state-variable constraint. It is found that the optimal trajectories may contain coasting arcs and that arcs lying along the state-variable constraint may be of intermediate thrust. A parametric study is made using a presently proposed space tug as the rescue vehicle. The results indicate that minimum-time trajectories could be satisfactorily flown by the tug in a low earth orbit rescue effort.

Introduction

THE successful completion of Skylab has demonstrated that long-duration manned space flight is not only feasible, but also highly productive. As the space shuttle orbiter is perfected and manned missions become more frequent and complex, the possibility of a malfunction or mishap leading to an emergency situation requiring space rescue will be substantially increased. If the emergency were critical, the time of flight of the rescue vehicle would become the overriding factor in the rescue attempt.

Unfortunately, the investigation of minimum-time trajectories with a thrust-limited rocket which could be adapted to a space rescue effort has been generally neglected. Minimum-fuel, continuous thrust rendezvous trajectories between widely separated spacecraft in the same circular orbit were investigated by Hahn and Itzen[1] and by Burrows and Wood[2] using nonlinear equations of motion. Since they considered a rocket operating at a constant and continuous thrust, their solutions are in fact minimum-time trajectories. The one-dimensional, minimum-time rendezvous problem has been solved analytically by Anderson, Falb, and Robinson.[3] They found that by specifying the final rocket mass instead of the initial rocket mass, the resulting optimal trajectory contained a period of zero thrust and the flight time was reduced. This one-dimensional problem was extended by Anderson and Othling[4] to the realistic case where the initial mass is fixed and a specified amount of fuel is available. Minimum-time transfer trajectories between coplanar orbits were studied by Anderson, Hamlin, and Othling.[5] They considered the fixed final mass boundary conditions and found that coasting arcs were present in the optimal solution as was true in the one-dimensional case. The possibility of using the space shuttle as a rescue vehicle has been investigated extensively.[6,7] These studies point out that the basic space shuttle has a very limited on-orbit maneuver capability due to weight restrictions, and methods of extending the utility of the shuttle as a rescue vehicle by augmenting the fuel or by means of a space tug have been proposed. The present investigation extends the variable-thrust, minimum-time rendezvous problem to three dimensions while adding the complexity of a state-variable constraint.

Received Oct. 6, 1975; revision received Jan. 16, 1976.

Index categories: Navigation, Control, and Guidance Theory; Earth Orbital Trajectories.

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Statement of the Optimization Problem

The mathematical problem to be solved is the determination of the optimal thrust program enabling an orbiting rescue vehicle to rendezvous with a distressed spacecraft in minimum time. The spacecraft in distress, which will be referred to as the target, is assumed to be passive and in a circular orbit. The rescue vehicle, which will be referred to as the interceptor, is also assumed to be in a circular orbit, not necessarily coplanar with the target nor at the same altitude.

The rocket engine of the interceptor is assumed to have a constant exhaust velocity and a thrust level which is controllable through the mass flow rate between zero and a specified upper limit. It is also assumed that the thrust vector can be aligned in any desired direction.

A minimum altitude is specified below which the interceptor cannot descend. This minimum altitude introduces a state-variable constraint into the optimization problem, but restricts the interceptor to altitudes where the effects of atmospheric drag can be neglected.

Various restrictions could be placed on the initial and final masses of the interceptor and on the fuel available. Here, the initial mass and fuel available are specified. This corresponds to the realistic case of a vehicle in some given configuration suddenly called upon to carry out a space rescue mission.

Coordinate System

The coordinate system is an Earth-centered, rotating, spherical coordinate system with the fundamental plane being the plane of the target orbit. The distance r is measured from Earth center to the interceptor. The radial ($\theta = 0$, $\phi = 0$) passes through the target vehicle; hence, the coordinate system rotates at the constant angular velocity ω of the target. The coordinate angle θ is measured in the fundamental plane and is positive in the direction of rotation of the target. The angle ϕ is measured in a plane perpendicular to the plane of the target orbit and is positive in the northern hemisphere. This rotating system was chosen because it greatly simplifies the transversality and terminal boundary conditions which, in turn, results in much more rapid convergence of the associated boundary-layer-value problem.

The thrust vector acting on the interceptor is determined by the three control variables, T, α, and β. The thrust magnitude T can vary between zero and its maximum value T_m. The thrust angle α is measured from the plane which is perpendicular to the radius vector and which passes through the interceptor. When α is positive, the thrust is upward attempting to force the interceptor away from Earth center. The thrust angle β is measured from the positive θ coordinate direction in the plane perpendicular to the radius vector.

Subject to the conditions given above, the nonlinear equations governing the system are

$$\dot{r}=v_r \quad \dot{\theta}=v_\theta/r\cos\phi \quad \dot{\phi}=v_\phi/r$$

$$\dot{v}_r=\frac{(v_\theta+r\omega\cos\phi)^2}{r}+\frac{v^2_\phi}{r}-\frac{\mu}{r^2}+\frac{T}{m}\sin\alpha$$

$$\dot{v}_\theta=-\frac{v_r v_\theta}{r}+\frac{v_\theta v_\phi}{r}\tan\phi-2v_r\omega\cos\phi$$

$$+2v_\phi\omega\sin\phi+\frac{T}{m}\cos\alpha\cos\beta$$

$$\dot{v}_\phi=-\frac{v_r v_\phi}{r}-\frac{(v_\theta+r\omega\cos\phi)^2}{r}\tan\phi+\frac{T}{m}\cos\alpha\sin\beta$$

$$\dot{m}=-T/c \tag{1}$$

where μ is the Earth gravitational constant, m the interceptor mass, and c the rocket exhaust velocity.

The previous equations are of the form $\dot{x}=f(x, u, t)$ where x is the vector of state variables and u is the vector of control variables. Therefore, the adjoint equations can be found through the relation $\dot{\lambda}=-\partial H/\partial x$ where $H=\lambda^T f$ and λ is the vector of adjoint variables. Since the rendezvous is to take place in minimum time, the index of performance (IP) for the optimization problem is the final time, t_f.

The State-Variable Constraint

The interceptor is prohibited from descending below a minimum altitude designated as $r_{\min}$. This restriction can be expressed as the state-variable inequality constraint $S=r_{\min}-r\leq 0$. When the interceptor is at the minimum altitude, the equal sign applies and the resulting state-variable equality constraint is $S=r_{\min}-r=0$. Differentiating this equality constraint until the control appears explicitly, and setting each derivative to zero, results in the relations which must hold when the interceptor is at the minimum altitude. These are

$$r=r_{\min} \quad v_r=0$$

$$T\sin\alpha=m\left[\frac{\mu}{r^2}-\frac{(v_\theta+r\omega\cos\phi)^2}{r}-\frac{v_\phi^2}{r}\right] \tag{2}$$

The latter relation serves as a control-variable equality constraint any time the trajectory lies on the state-variable boundary. Hence, the state equations on the boundary are

$$\dot{r}=0 \quad \dot{\theta}=v_\theta/r\cos\phi \quad \dot{\phi}=v_\phi/r \quad \dot{v}_r=0$$

$$\dot{v}_\theta=(v_\theta v_\phi/r)\tan\phi+2v_\phi\omega\sin\phi+\frac{T}{m}\cos\alpha\cos\beta$$

$$\dot{v}_\phi=-[(v_\theta+r\omega\cos\phi)^2/r]\tan\phi+\frac{T}{m}\cos\alpha\sin\beta$$

$$\dot{m}=-\frac{T}{c} \quad \text{where } r=r_{\min} \tag{3}$$

and $\alpha=\sin^{-1}\dfrac{m}{T}\left[\dfrac{\mu}{r^2}-\dfrac{(v_\theta+r\omega\cos\phi)^2}{r}-\dfrac{v_\phi^2}{r}\right]$.

In the preceding formulation, the control angle α has become a function of the thrust magnitude and the state. Hence, the only independent control variables are T and β. The adjoint equations on the boundary can be found from the above state equations through $\dot{\lambda}=-\partial H/\partial x$.

Boundary Conditions and Transversality Relationships

The initial position of the interceptor is specified by four parameters: the distance from Earth center r_I; the phase angle from the target to the interceptor θ_I; the angle of inclination of the interceptor orbital plane from that of the target γ; and the angle to the interceptor from its ascending node measured in the target plane θ_{AI}. These parameters are shown in Fig. 1. Initial values of the state variables are found in terms of these parameters. The results are

$$r(0)=r_I \quad \theta(0)=\theta_I$$

$$\phi(0)=\tan^{-1}(\tan\gamma\sin\theta_{AI}) \quad v_r(0)=0$$

$$v_\theta(0)=\left(\frac{\mu}{r_I}\right)^{1/2}\frac{\cos\gamma}{\cos\phi(0)}-r_I\omega\cos\phi(0)$$

$$v_\phi(0)=\left(\frac{\mu}{r_I}\right)^{1/2}\sin\gamma\cos\theta_{AI} \quad m(0)=m_0 \tag{4}$$

The mission of the interceptor is to rendezvous, or match, the position and velocity of the target. Therefore,

$$r(t_f)=r_T \qquad \phi(t_f)=0 \qquad v_\theta(t_f)=0$$

$$\theta(t_f)=0 \qquad v_r(t_f)=0 \qquad v_\phi(t_f)=0 \tag{5}$$

The amount of fuel that can be burned during the mission is specified. If all allowable fuel is burned, the final mass m_f of the interceptor is known. If, however, the rendezvous can be completed without using all the fuel, then the final mass is free. In the latter case, the transversality relationships require that $\lambda_m(t_f)=0$. So, two cases arise depending on the fuel available. They are

$$m(t_f)=m_f \quad \text{(all fuel used)}$$

and

$$\lambda_m(t_f)=0 \quad \text{(all fuel not used)} \tag{6}$$

Since the index of performance of the optimization problem is the final time t_f, transversality relationships require that $H(t_f)=1$.

At the time t_I, when going on the boundary, the conditions $r(t_I)=r_{\min}$ and $v_r(t_I)=0$ must be satisfied, and the adjoint variables have discontinuities governed by

$$H(t_I^+)=H(t_I^-) \quad \lambda_r(t_I^+)=\lambda_r(t_I^-)+\nu_r$$

$$\lambda_{v_r}(t_I^+)=\lambda_{v_r}(t_I^-)+\nu_{v_r}$$

Thus, all conditions necessary for the solution of the multipoint boundary-value problem have been determined. Analysis of the maximum principle yields the optimal control off the state-variable boundary.

$$\sin\beta^*=\lambda_{v_\phi}/D_I \qquad \cos\beta^*=\lambda_{v_\theta}/D_I \tag{7}$$

$$\sin\alpha^*=\lambda_{v_r}/D \qquad \cos\alpha^*=D_I/D \tag{8}$$

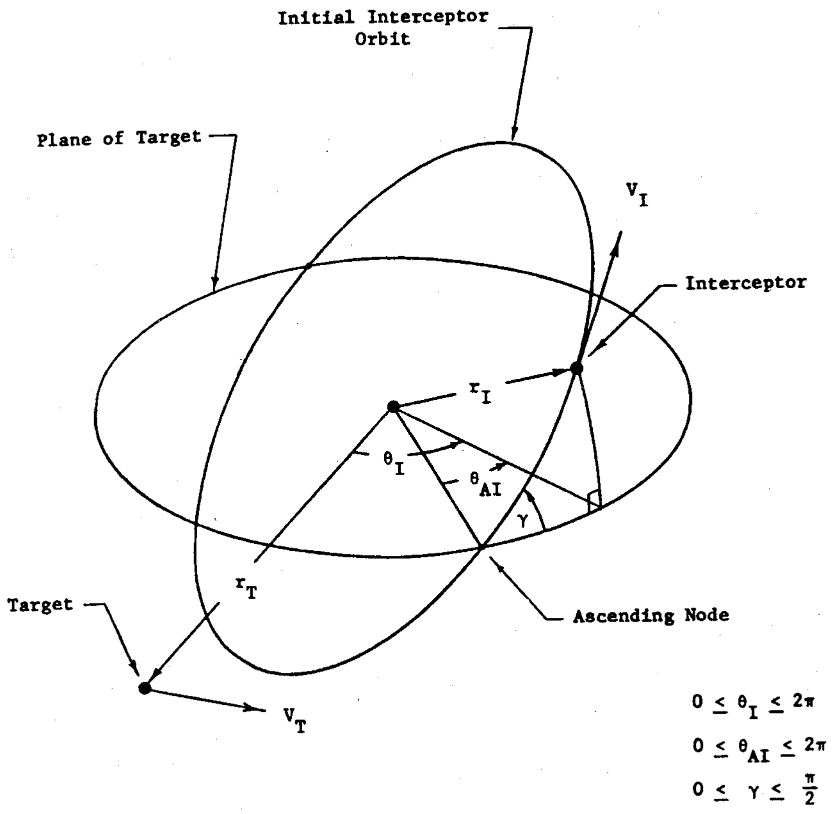

Fig. 1 Initial vehicle positions.

where $D=(\lambda^2_{v_r}+\lambda^2_{v_\theta}\lambda^2_{v_\phi})^{1/2}$ and $D_I=(\lambda^2_{v_\phi}+\lambda^2_{v_\phi})^{1/2}$

$$T^*=T_m \text{ if } \kappa>0 \quad T^*=0 \text{ if } \kappa<0 \tag{9}$$

where $\kappa=D/m-\lambda_m/c$ serves as a thrust switching function.

When the trajectory lies on the state-variable boundary, the optimal control angle β^* is given by Eq. (7). However, α^* and T^* depend on the value of the term $cD_I/m\lambda_m$. The rule is:

a) if $cD_I/m\lambda_m \geq 1$, $T^*=T_m$, $\alpha^*=\sin^{-1}(f_c/T_m)$

where

$$f_c=m\left[\frac{\mu}{r^2}-\frac{(v_\theta+r\omega\cos\phi)^2}{r}-\frac{v_\phi^2}{r}\right]$$

b) If $cD_I/m\lambda_m<1$, $\alpha^*=\cos^{-1}(cD_I/m\lambda_m)$,

$$T^*=f_c/\sin\alpha^*$$

c) If the case b) the calculated value of T^* is greater than T_m,

$$T^*=T_m, \quad \alpha^*=\sin^{-1}(f_c/T_m)$$

In case b), the optimal thrust has an intermediate value. This intermediate thrust is not due to singular control, but results from applying the maximum principle on the state-variable boundary.

Method of Solution

The resulting multipoint boundary-value problem was solved using the indirect shooting technique with a modified Newton's method for convergence.[8, 9] Estimates were made of the final time and the unknown initial values of the adjoint variables. In constrained problems, it was also necessary to estimate the time (or times) the trajectory went on the boundary and the magnitude of discontinuities in the adjoint variables which occurred at that time. The time coming off the boundary was not estimated since it was determined when the unconstrained optimal control resulted in the trajectory leaving the boundary.

The system and adjoint equations were integrated numerically on a CDC 6500 computer using a 4th order Bashforth Moulton multistep predictor corrector integration scheme with a 4th order Runge Kutta procedure to compute starting values. The necessity for numerically integrating the coasting arcs was eliminated by using the method of subarc elimination proposed by Vincent and Mason[10] and implemented by Thurneck.[11] The coast times of the eliminated arcs were of course included in the calculation of t_f.

In some cases, the integration was halted when one of the terminal conditions was satisfied. If so, it was not necessary to estimate the final time, and the dimension of the problem was reduced. Also, the estimate of the intermediate time was sometimes eliminated by use of one of the intermediate conditions.

Optimal values from a nearby trajectory were used as initial estimates in the first integration with a new set of data. This continuation process proceeded to new data points until the desired results were obtained. Convergence was almost always obtained within two to six iterations, and only in extreme cases with severe constraints was oscillatory behavior observed. Each iteration required an average of 15 sec, and most trajectories converged within one min of computer time.

The computer program was not designed to transition automatically from an unconstrained to a constrained trajectory. The constrained trajectory of course required estimates of intermediate times and jumps in the adjoint variables. When the constraint was encountered, it was necessary to manually select a new subroutine taking the new conditions into account.

Characteristics of Minimum-time Rescue Trajectories

Two-dimensional, unconstrained rescue trajectories are plotted in Figs. 2 and 3. The conditions used to generated these trajectories are listed in Table 1. All of the trajectories are of the thrust-coast-thrust type. Both vehicles are moving counterclockwise. The two small circles indicate the initial and final positions of the target, and the stars indicate the

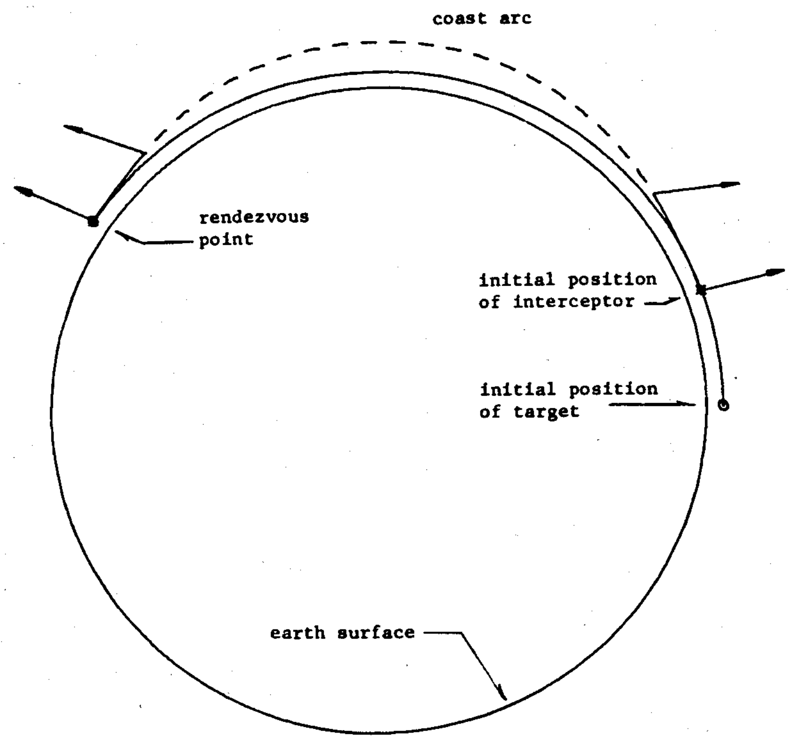

Fig. 2 Rendezvous for $\theta_I = -20°$.

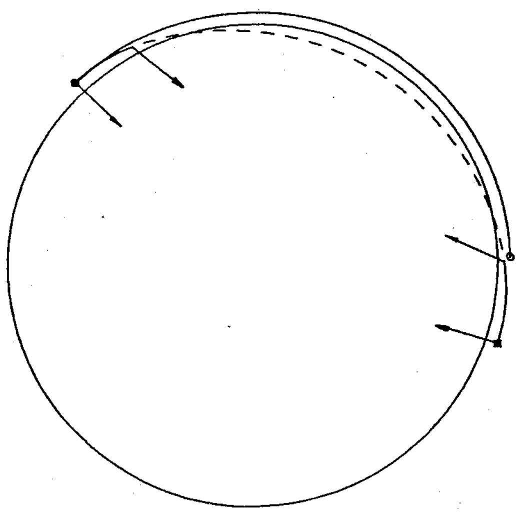

Fig. 3 Unconstrained rendezvous for $\theta_I = -20°$.

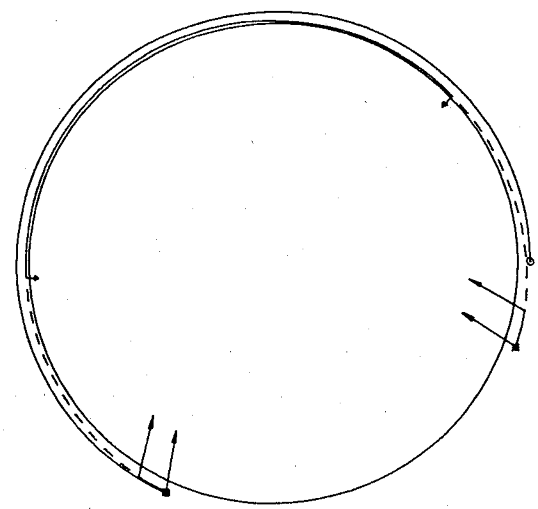

Fig. 4 Constrained rendezvous for $\theta_I = -20°$.

initial and final positions of the interceptor. The thrust acceleration vector at the beginning and end of each thrusting arc is also shown. The large solid circle represents the surface of the Earth.

In Fig. 2, the interceptor initially leads the target by 20°. The initial thrust is directed outward and slightly to the rear resulting in a high coasting arc allowing the interceptor to close with the target. The final thrust is directed outward and slightly forward resulting in the rendezvous. Figure 3 shows a rendezvous in which the interceptor initially trails the target by 20°. The initial thrust is down and slightly forward, and the optimal unconstrained trajectory is subterranean, which of course is not admissible.

If a minimum altitude of 43.4 nautical miles is imposed on the -20° trajectory of Fig. 3, the constrained trajectory of Fig. 4 results. This trajectory possesses an intermediate thrust arc which lies along the minimum altitude constraint. The two small arrows indicate the thrust acceleration at the beginning and end of the intermediate thrust arc. As expected, the time to complete the rendezvous is considerably increased when the altitude restriction is present.

Behavior of the Thrust Switching Function

The behavior of the thrust switching function κ is demonstrated in Fig. 5 for a constrained trajectory such as the one depicted in Fig. 4. At the point where the trajectory goes on the boundary, discontinuities occur in the adjoint variables λ_r and λ_{v_r}. As a result, both the switching function and its slope become discontinuous at that point. If the altitude constraint were raised higher, these discontinuities would become larger, and the length of the intermediate thrust arc would increase. Of course, as the altitude constraint is raised, the time to rendezvous is increased.

Effect of the Altitude Constraint on Continuous Thrust Trajectories

If there is excess fuel available, the switching function remains positive, and the trajectory is of the continuous maximum thrust type. If the minimum altitude constraint is then raised in increments, a series of trajectories similar to those of Fig. 6 results.

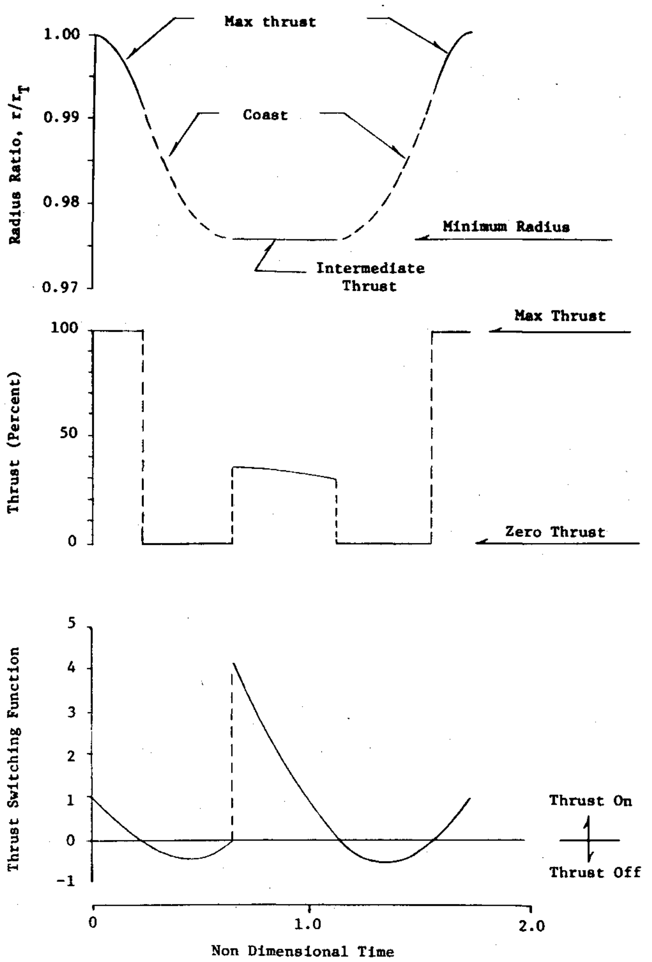

Fig. 5 Behavior of thrust switching function for trajectory with constrained arc.

Table 1 Conditions for trajectories of Figs. 2-4[a]

For All Cases –	
Initial target altitude	174 nautical miles (322 km)
Initial interceptor altitude	174 nautical miles (322 km)
Initial interceptor weight	125,000 lb (56,700 kg)
Maximum thrust	50,000 lb (222,000 N)
Specific impulse	500 sec
Final interceptor weight	75,000 lb (34,000 kg)

Figure	Initial phase angle	Time to rendezvous
2	20	2189 sec
3	-20	1988 sec
4	-20	3687 sec

[a]For Fig. 4, the minimum altitude constraint is 43.4 nautical miles (80.4 km).

Table 2 Proposed rescue tug characteristics (approximate values)

Gross weight	62,000 lb (28,120 kg)
Burn out weight (with rescue module)	16,500 lb (7,480 kg)
Thrust	20,000 lb (89,000 N)
Specific impulse	470 sec
Rescue module weight	10,000 lb (4,540 kg)
Rescue fuel weight	45,500 lb (20,640 kg)
ΔV capability	20,000 ft/sec (6,100 m/sec)

Table 3 Typical rescue trajectory

Target altitude	300 nautical miles (556 km)
Initial interceptor altitude	200 nautical miles (370 km)
Target-interceptor phase angle	20 deg
Initial interceptor orbit inclination	10 deg
Minimum altitude constraint	50 nautical miles (93 km)
Rendezvous time	1586 sec

Figure 6a is an unconstrained trajectory. Figure 6b is a constrained trajectory which touches the state-variable boundary at a single point. In this trajectory, the adjoint variable λ_r is discontinous at the point where the trajectory touches the boundary. Also, as the boundary is raised, an inflection develops in the last half of the trajectory.

In Fig. 6c, the trajectory has a short arc lying on the state-variable boundary. In this case, the adjoint variables λ_r and λ_{v_r} both have discontinuities at the point where the trajectory goes on the boundary.

The trajectory of Fig. 6d is very severely constrained. The arc lying along the boundary is longer and the inflection which formed has caused the trajectory to come back and touch the boundary a second time. An additional discontinuity occurs in the adjoint variable λ_r at this latter junction point.

The reason for the inflection in these constrained trajectories can be explained in general terms by considering the relative angular velocity between the interceptor and the target. To effect a minimum-time rendezvous, this relative angular velocity should be, on the average, as large as possible. A large angular velocity can be brought about by either increasing the tangetial velocity or by decreasing the radius. All of the trajectories exhibit the tendency to quickly decrease the altitude. When a restrictive constraint is placed upon the radius, maintaining a high relative tangential velocity through the central portion of the trajectory becomes more advantageous than maintaining the minimum altitude. Hence, a "hump" develops which is evident in Fig. 6d. During this period, the optimal control allows the trajectory to temporarily leave the state-variable constraint in order to increase the tangential velocity.

Three-Dimensional Rescue Trajectories

The discussion to this point has been limited to two-dimensional trajectories. Three-dimensional trajectories, however, possess essentially the same characteristics, especially as to the behavior of the thrust switching function and the inflections which develop in constrained trajectories. The only significant difference is an out-of-plane component of the thrust vector. A parametric study of three-dimensional trajectories, including plane change maneuvers, is included in the Ph.D. thesis by the first author,[12] from which this paper was condensed.

Capabilities of a Shuttle Launched Tug as a Space Rescue Vehicle

The feasibility of using the space shuttle orbiter as an emergency rescue vehicle is being investigated, as indicated above. Several plans have been proposed to increase the on-orbit capability of the shuttle, including a cargo bay propellant tank, orbital refueling, or a shuttle launched tug. A single space tug would have a ∇V capability of over 20,000 ft/sec (6,100 m/sec), and two tugs in a tandem configuration would have a capability of 29,000 ft/sec (8,850 m/sec) which is sufficient for lunar round trip. If the tug were in a low earth orbit rescue role, it would have the ability to make large plane and phase angle changes and would considerably hasten the rescue effort.

The purpose here is to determine the capabilities of the space tug in its presently proposed configuration as a low earth orbit rescue vehicle using minimum-time trajectories. A

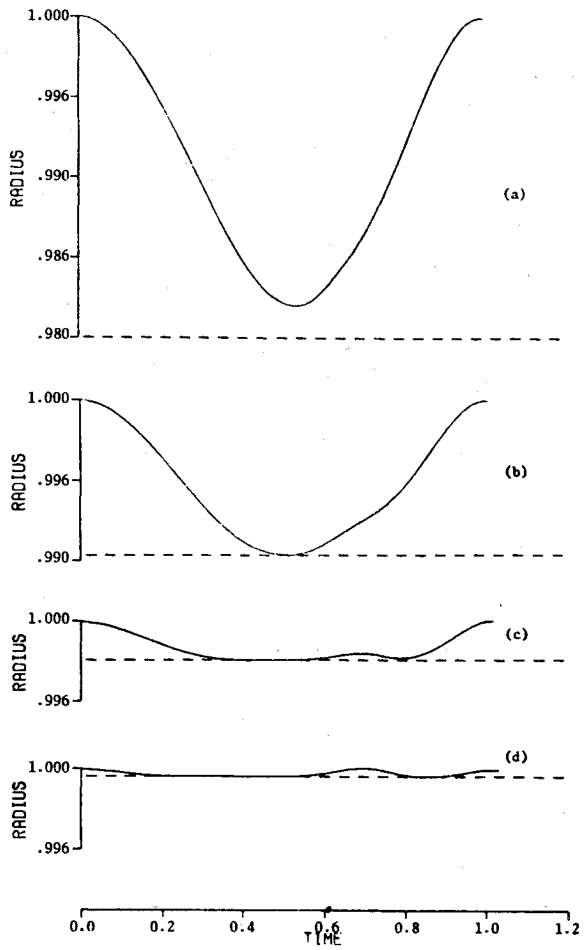

Fig. 6 Effect of altitude constraint on continuous thrust trajectories.

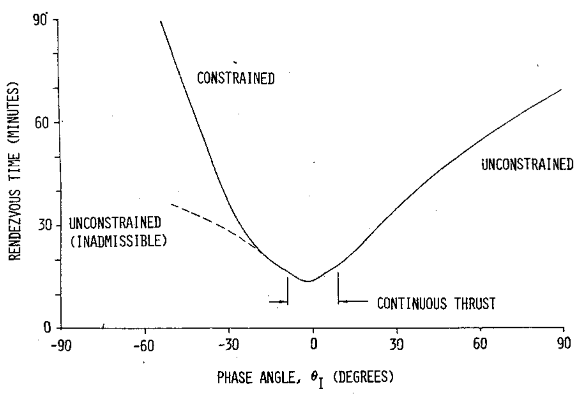

Fig. 7 Effect of target-interceptor phase angle on rendezvous time.

parametric study was made using the proposed space tug performance shown in Table 2. With this performance, a typical rescue trajectory was generated. The trajectory data is listed in Table 3.

Figures 7 and 8 demonstrate the capability of the rescue tug to perform a minimum-time rendezvous in low earth orbit. Figure 7 shows how the rendezvous time varies with the target-interceptor phase angle, θ_I. Above approximately $\theta_I = 9°$, trajectory is of the thrust-coast-thrust type depicted in Fig. 2. Between $\theta_I = -9°$ and $+9°$, there is excess fuel available, and the trajectory is continuous thrust. Between $\theta_I = -9°$ and $-17°$ the trajectory is again of the thrust-coast-thrust type. The dashed curve below $\theta_I = -17°$ indicates unconstrained trajectories which violate the minimum altitude constraint. If the trajectory is constrained, an intermediate thrust arc develops as in Fig. 4. In this case, the rendezvous

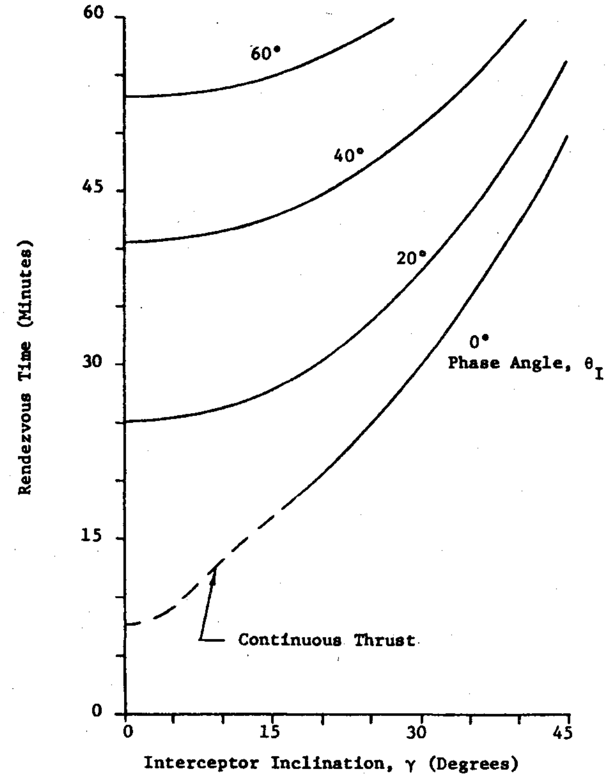

Fig. 8 Effect on interceptor inclination and phase angle on rendezvous time.

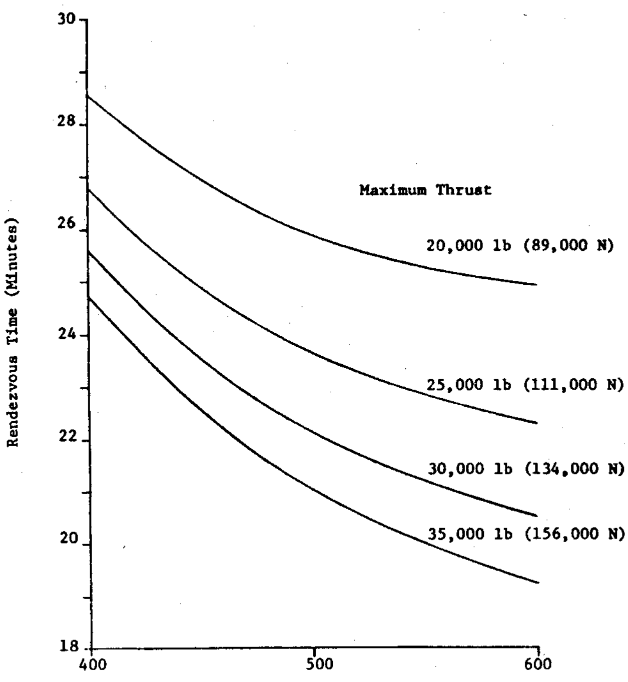

Fig. 9 Effect on specific impulse and maximum thrust on rendezvous time.

time increases rapidlyy as the magnitude of the phase angle increases as indicated by the solid curve below $\theta_I = -17°$.

The effect of the initial inclination of the interceptor orbit on the rendezvous time for various phase angles is demonstrated in Fig. 8. As expected, the time increases rapidly as the initial angle between the orbital planes is increased. For small phase angles and inclinations, the fuel available becomes excessive and continuous thrust trajectories result. These are indicated by the dashed line on the $\theta_I = 0$ curve. Figure 9 indicates the decrease in rendezvous time which would result from increasing the thrust or specific impulse of the rocket engine.

Summary

The trajectories investigated here involve large phase angle and plane changes by the interceptor resulting in rendezvous with the passive target vehicle. During the development of the mathematical problem, it is found that discontinuities occur in the adjoint variables at points where the trajectory goes on the state-variable boundary. Application of the maximum principle yields the optimal control angles for the thrust vector and reveals the existence of a thrust switching function determining thrust cutoff and relight points. It also shows that intermediate thrust levels may at times be optimal for arcs lying along the state-variable constraint.

The optimal trajectories resulting from the analysis are found to exhibit varied characteristics. Free trajectories are either maximum thrust – coast – maximum thrust or continuous maximum thrust depending upon the amount of fuel available. One type of constrained trajectory is maximum thrust – coast – intermediate thrust – coast – maximum thrust. In this case, the intermediate thrust arc lies along the state-variable constraint. When the constrained trajectory is the continuous thrust type, the thrust remains maximum throughout. When the state-variable constraint is severe, the trajectory may leave the boundary temporarily and return a second time.

A parametric study is made using a presently proposed space tug as the rescue vehicle. It is demonstrated that the tug can make a successful rendezvous involving large phase angle and plane changes in a relatively short time.

In conclusion, it has been demonstrated that minimum-time trajectories are practical and should be seriously considered for space rescue missions. If a space rescue vehicle and its control software were designed with more emphasis on reducing the time to achieve rescue, the overall effectiveness of rescue mission profiles would be substantially upgraded. Moreover, the effort would add immeasurably to the safety of our astronauts in the hostile environment of outer space.

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